OCR RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(A)

Pure Mathematics 3

Section A

Friday **18 JANUARY 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 3 printed pages and 1 blank page.

1 **(a)** Find $\int_0^{\frac{1}{2}\pi} x \sin x \, dx$. [4]

(b) Differentiate $\sin^3 2x$ with respect to x. [3]

(c) Express $2\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where α is in degrees. Give the values of R and α correct to 1 decimal place. [3]

(d) Write down small-angle approximations for $\sin h$ and $\cos h$. Hence show that, for small values of h,

$$\frac{\sin(x+h) - \sin x}{h} \approx \cos x - \tfrac{1}{2} h \sin x.$$

What does this result suggest as $h \to 0$? [5]

[Total 15]

2 **(i)** Express $\frac{1-x}{(1+x)(1+x^2)}$ in the form $\frac{A}{1+x} + \frac{Bx+C}{1+x^2}$. [4]

(ii) Hence show that the solution of the differential equation

$$\frac{dy}{dx} = \frac{y(1-x)}{(1+x)(1+x^2)},$$

given that $y = 1$ when $x = 0$, is

$$y = \frac{1+x}{\sqrt{1+x^2}}.$$ [7]

(iii) Find the first three terms of the binomial expansion of $\frac{1}{\sqrt{1+x^2}}$. Hence find a polynomial approximation for $y = \frac{1+x}{\sqrt{1+x^2}}$. up to the term in x^5. [4]

[Total 15]

3 With respect to coordinate axes Oxyz, A is the point (2, 0, 0), B is (0, 0, 1) and C is (3, 1, 3).

(i) Find the vectors $\overrightarrow{CA}$ and $\overrightarrow{CB}$. Hence find angle ACB. [4]

(ii) Write down the cartesian equation of the plane p through A with normal vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

Verify that B also lies in this plane. [3]

(iii) Write down the vector equation of the line through C perpendicular to the plane p.

Find the point of intersection of this line with the plane, and the distance from C to the plane. [7]

[Total 14]

4 Fig. 4 shows a sketch of the curve with equation $y^2 = (1 - 2x)^3$. The curve meets the x-axis at A and crosses the y-axis at the points B and C.

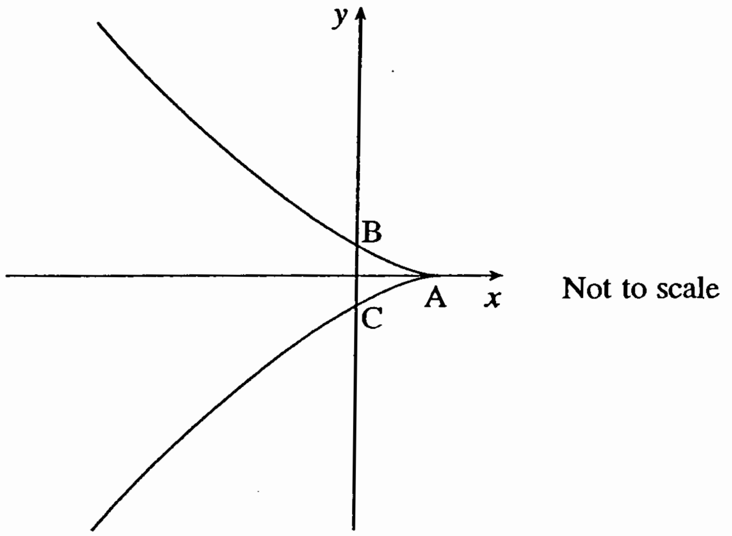

Fig. 4

(i) Find the coordinates of the points A, B and C. [3]

(ii) Show that the gradient of the curve at the point B is –3. [5]

(iii) Verify that

$$x = \tfrac{1}{2}\left(1 - t^2\right), \qquad y = t^3$$

are parametric equations for the curve.

Find $\dfrac{dy}{dx}$ in terms of t, and show that the equation of the tangent to the curve at the point with parameter t is

$$6tx + 2y + t^3 - 3t = 0.$$ [8]

[Total 16]

Planetary Systems

Introduction

During most of the 18th century, just five planets, apart from Earth, were known: Mercury, Venus, Mars, Jupiter and Saturn.

In 1766, Titius of Wittenburg discovered that the distances from the Sun of the known planets formed an interesting mathematical sequence. This was published six years later by Johann Bode and became known as Bode's Law. Shortly afterwards, in 1781, the planet Uranus was first observed and it too obeyed Bode's Law.

Before going on to look at Bode's Law, two points are worth noting.

(i) In this article, distances are given in metric units and so the numbers differ by a constant scale factor from those which were used in the 18th century.

(ii) The paths of planets are ellipses rather than circles. Fig. 1 shows such a path (or orbit) around a star. Notice that the star is not at the centre of the ellipse.

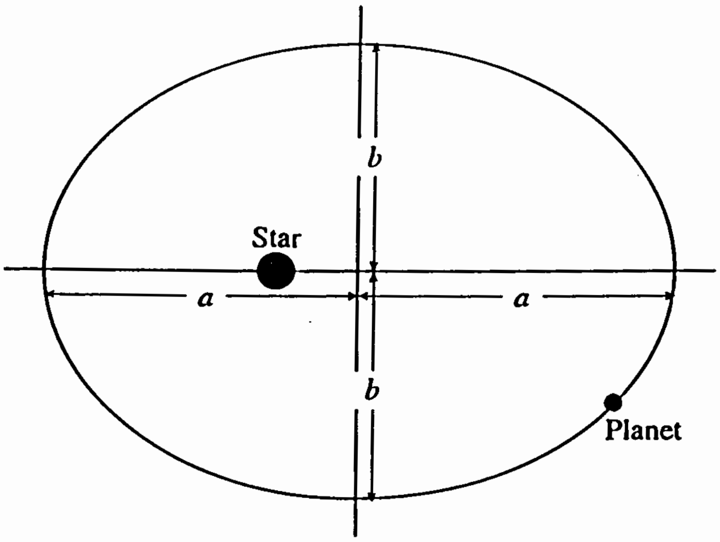

Fig. 1

A measure of how much an ellipse differs from a circle is called its *eccentricity*. The eccentricity of an ellipse is given by e ($0 \leqslant e < 1$) in the equation

$$e^2 = 1 - \frac{b^2}{a^2}$$

where a and b are the lengths shown in Fig. 1. A circle has eccentricity 0.

With the exception of Pluto, the planets in our solar system have nearly circular orbits. Consequently, in this article the eccentricity is ignored. The orbits are taken to be circular with the Sun at the centre.

Bode's Law

Take the geometric progression 1, 2, 4, … , 64 and introduce 0 as a first term:

$$0, 1, 2, 4, \ldots, 64.$$

Now multiply each term by $4\frac{1}{2}$ and then add 6 to obtain Bode's numbers:

$$6, 10.5, 15, 24, \ldots, 294.$$

These numbers, multiplied by 10^7, give good approximations for the distances in kilometres of the planets Mercury to Uranus from the Sun, as shown in Table 2.

	Planet	Bode's number	Distance from the Sun ($\times 10^7$ km)
1	Mercury	6	5.8
2	Venus	10.5	10.8
3	Earth	15	15.0
4	Mars	24	22.8
5		42	
6	Jupiter	78	77.8
7	Saturn	150	142.7
8	Uranus	294	286.9

Table 2

It was immediately noticed that there seemed to be a planet missing, and a group of astronomers were assigned the task of finding it; they were nicknamed "The Celestial Police". In the early years of the 1800s four bodies, rather than one, were found at about the distance from the Sun predicted by Bode's Law. All of these were small and they became known as *asteroids*. Since then several thousand asteroids have been discovered, many of them little more than large rocks, occupying a belt between Mars and Jupiter.

In 1841, the planet Neptune was discovered at a distance of 449.8×10^7 km from the Sun, a distance which did not agree with that predicted by extending Bode's Law.

In 1930, Pluto was discovered and it also failed to conform to Bode's Law. However, Pluto differs from the other planets in so many respects that most astronomers no longer call it a planet. As far as this article is concerned, it is reasonable to ignore it.

The nature of Bode's Law

Is Bode's Law just a fluke result which happens to work for the first seven planets and the asteroid belt? Or does it encapsulate some underlying truth about the formation of our solar system, Neptune excepted?

It is important to realise that Bode's Law is not a law in the mathematical sense. It provides an approximate, but not exact, description of the positions of some of the planets. It is based only on observation, not on any theoretical considerations.

However, it is often the case that experimental observations lead to theoretical understanding. The next stage is to form a model to explain the observations. If it is a good model it can then be used to predict other results.

So for Bode's Law to be of any value, it must be supported by a theoretical model for the formation of our solar system. It is also possible that such a model could be refined to explain the anomaly of Neptune.

A single counter-example is all that is needed to disprove a mathematical law. The situation is not quite so definite for a model; it may still have some value as a partial explanation of the situation.

Alternatives to Bode's Law

Bode's Law may be written as

$$R_1 = 6$$

$$R_n = 6 + (4.5 \times 2^{n-2}) \qquad \text{for } n \geqslant 2,$$

where R_n is the distance of the nth planet from the Sun.

There are several arbitrary constants in this. The numbers 6, 4.5 and 2 have all, in effect, been picked to fit the data. The type of formula has also been chosen to match the data. There are many other types of formula that could have been tried; maybe Titius did so before finding one that worked well.

You can fit the data for any 8 planets exactly using a polynomial formula with 8 unknowns, for example

$$R_n = a + bn + cn^2 + dn^3 + en^4 + fn^5 + gn^6 + hn^7.$$

Substituting for Mercury ($n = 1$) gives

$$5.8 = a + b + c + d + e + f + g + h$$

and for Venus ($n = 2$) gives

$$10.8 = a + 2b + 4c + 8d + 16e + 32f + 64g + 128h$$

and so on. You end up with 8 equations in 8 unknowns and so an exact solution is theoretically possible.

However, you do not actually need anything like as many unknowns to produce the sort of accuracy that Bode's Law achieved. Take, for example, the polynomial with 3 unknowns

$$R_n = an + bn^3 + cn^5.$$

Substitute in the data for Venus, Jupiter and Uranus. You find:

$$a = 5.849\ldots \qquad b = -0.1510\ldots \qquad c = 0.009687\ldots.$$

The predictions from this formula are given in Table 3. Those for Venus, Jupiter and Uranus are of course correct.

Planet	Prediction	Distance from the Sun ($\times 10^7$ km)
Mercury	5.7	5.8
Venus	10.8	10.8
Earth	15.8	15.0
Mars	23.7	22.8
Asteroids	40.6	
Jupiter	77.8	77.8
Saturn	152.0	142.7
Uranus	286.9	286.9

Table 3

The point is not that there is something special about the polynomial formula we have found, but that it is not difficult to find a different formula which fits the data at least as well as Bode's Law.

So Bode's Law could indeed be just a fluke.

Models for Planetary Systems

Bode's Law was a first attempt to provide information about the distribution of planets within a system. While the particular form proposed in the 18$^{\text{th}}$ century may prove to be of no more than historical interest, the underlying problem is one of the key issues which any theory (or model) for the evolution of a planetary system must explain.

At the moment, there are two main types of theory of how the solar system formed.

According to the *Solar Nebula Theory*, the Sun condensed from a cloud of gas. As it contracted under its own gravity, it spun faster and faster, like an ice-skater. Eventually it threw off a ring of material which then formed into the planets with nearly circular orbits. Although this theory is widely held, there are major problems associated with it. For example, it predicts that the Sun should be spinning much faster than it actually is.

In the *Capture Theory*, the planets were formed from material captured from a passing cloud of gas. This occurred at a time when both the Sun and the passing cloud were condensing from a much larger cloud of gas. Supporters of this theory claim that it can explain all the major features of the solar system. Planetary orbits would initially be highly elliptical but the effect of gravitational forces between the planets would be to make their orbits progressively more nearly circular.

The origins of the planets are completely different according to these two theories. In one, the planets were formed from material from within the Sun; in the other, the material came from elsewhere.

At the moment, we have only one reasonably complete set of data, that from our own solar system, by which we can compare different theories. However, the situation is changing. At the time of writing this article, over 60 stars have been found to have an accompanying planet. In 1999, for the first time, a star (υ *Andromeda*) was discovered to have at least three planets. All of the planets found so far are large, at least the size of Jupiter; techniques are not yet available to detect small or medium-sized planets.

However we can reasonably hope that in a few years it will be possible to find whole planetary systems. They may give evidence to support either theory – or even a totally new one.

The orbits of many of the planets that have been discovered are highly elliptical. This may be taken as evidence in favour of the Capture Theory.

For Examiner's Use

1 Justify the statement "A circle has eccentricity 0" on line 17.

....

....

.... [2]

2 Justify the statement in the paragraph starting at line 33 that the distance of Neptune from the Sun does not agree with that predicted by extending Bode's Law.

....

....

....

.... [3]

3 Bode's Law is formed using the sequence in line 22,

$$0, 1, 2, 4, \ldots., 64.$$

State the number which should replace 0 as the first term if the whole sequence is to be a geometric progression.

Find the distance of Mercury from the Sun given by this new first term.

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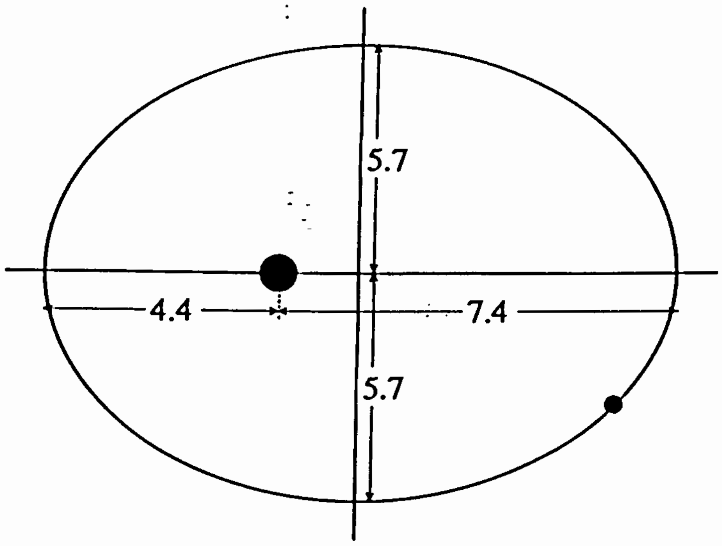
....

.... [3]

For Examiner's Use

4 The ellipse in the diagram below illustrates the orbit of Pluto. The distances are given in units of 10^9 km.



Calculate the eccentricity of Pluto's orbit.

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....

....

.... [2]

5 In the article, distances are given in units of 10^7 km, so that the distance of Earth from the Sun is given as 15. When the law was originally put forward the metric system did not exist and the distance of Earth from the Sun was called 10 units.

Give R_1 and the formula for R_n in these old units.

....

....

....

.... [2]

[Turn over

For Examiner's Use

6 The table below gives the mean distances of the five major moons of Uranus from the planet.

n	Moon	Distance R ($\times 10^4$ km)
1	Miranda	12.9
2	Ariel	19.1
3	Umbriel	26.6
4	Titania	43.6
5	Oberon	58.4

A student used three of the moons to obtain the approximate relationship

$$R = 13.66n - 0.7875n^3 + 0.02833n^5.$$

Which three moons did the student use?

....

....

....

.... [2]

7 The article ends "The orbits of many of the planets that have been discovered are highly elliptical. This may be taken as evidence in favour of the Capture Theory".

Justify the final statement.

....

.... [1]

Mark Scheme

1(a) $\int_0^{\pi/2} x\sin x\,\mathrm{d}x = \int_0^{\pi/2} x\frac{d}{dx}(-\cos x)\,\mathrm{d}x$ $= \left[-x\cos x\right]_0^{\pi/2} + \int_0^{\pi/2}\cos x\,\mathrm{d}x$ $= 0 + \left[\sin x\right]_0^{\pi/2}$ $= 1$	M1 A1 A1ft A1 [4]	Parts: $u = x$, $\frac{dv}{dx} = \sin x$ +some attempt to integrate sinx $[-x\cos x]$...... ...+$[\sin x]_0^{\pi/2}$ ft their v w.w.w.
(b) $y = \sin^3 2x$ let $u = \sin 2x, \Rightarrow y = u^3$ $\Rightarrow \frac{dy}{du} = 3u^2, \frac{du}{dx} = 2\cos 2x$ $\Rightarrow \frac{dy}{dx} = 3u^2.2\cos 2x$ $= 6\sin^2 2x\cos 2x$ Or use of double angle formula and product rule for differentiation	M1 DM1 A1cao M1 A1 A1cao [3]	chain rule. Attempt at $\frac{dy}{du}$ and $\frac{du}{dx}$ 2cos $2x$ or cos $2x$ Special case SC B1 for 6 $\sin^2 2x$ Use of formula for cos $4x$ and product rule Correct application of both Any correct expression
(c) $2\sin x - 3\cos x \equiv R\sin(x - \alpha)$ $\Rightarrow 2\sin x - 3\cos x \equiv R(\sin x\cos\alpha - \cos x\sin\alpha)$ $\Rightarrow R\cos\alpha = 2, R\sin\alpha = 3,$ $\Rightarrow R^2 = 2^2 + 3^2 = 13, R = \sqrt{13} = 3.6$ $\tan\alpha = 3/2, \Rightarrow \alpha = 56.3°$	M1 B1 A1 [3]	The identity s.o.i. (not implied by $\tan\alpha = \frac{2}{3}$ or similar.) Accept $\sqrt{13}$ Accept 0.983 radians
(d) $\sin h \approx h$, $\cos h \approx 1 - \frac{1}{2}h^2$ $[\sin(x + h) - \sin x]/h$ $= [\sin x\cos h + \cos x\sin h - \sin x]/h$ $\approx [\sin x(1 - \frac{1}{2}h^2) + \cos x.h - \sin x]/h$ $= [\sin x - \frac{1}{2}h^2\sin x + h\cos x - \sin x]/h$ $= \cos x - \frac{1}{2}h\sin x$ As $h \to 0$, $\cos x - \frac{1}{2}h\sin x \to \cos x$ So the derivative of $\sin x$ is $\cos x$.	B1 M1 M1 E1 B1	Both correct Use of compound angle formula substituting small angle approximations for sin h and cos h www

	[5] [15]	

2 (i) $\frac{1-x}{(1+x)(1+x^2)} \equiv \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$ $\Rightarrow 1-x \equiv A(1+x^2)+(Bx+C)(1+x)$ $x=-1 \Rightarrow 2=2A, A=1$ constants: $1 = A + C \Rightarrow C = 0$ coefft of x^2: $0 = A + B \Rightarrow B = -1$ so $\frac{1-x}{(1+x)(1+x^2)} \equiv \frac{1}{1+x} - \frac{x}{1+x^2}$	M1 B1 A1 A1 [4]	Identity s.o.i. plus attempt to equate coeffs or substitute a value of x $A=1$ $C=0$ $B=-1$ i.s.w. after the above results.
(ii) $\frac{dy}{dx} = \frac{y(1-x)}{(1+x)(1+x^2)}$ $\Rightarrow \int \frac{1}{y}dy = \int \frac{1-x}{(1+x)(1+x^2)}dx$ $\Rightarrow \ln y = \int (\frac{1}{1+x} - \frac{x}{1+x^2})dx$ $= \ln(1+x) - \frac{1}{2}\ln(1+x^2) + c$ When $x=0, y=1$ $\Rightarrow \ln 1 = \ln 1 - \frac{1}{2}\ln 1 + c \Rightarrow c = 0$ $\Rightarrow \ln y = \ln(1+x) - \frac{1}{2}\ln(1+x^2)$ $= \ln\frac{1+x}{\sqrt{1+x^2}}$ $\Rightarrow y = \frac{1+x}{\sqrt{1+x^2}}$ *	M1 M1 B1 A1ft A1ft M1 E1 [7]	separating variables substituting their partial fractions $\ln y = \ldots$ $\ln(1+x)\ldots$ $\ldots - \frac{1}{2}\ln(1+x^2)+c$ evaluating c. M0 following incorrect use of the rules of logs.
(iii) $\frac{1}{\sqrt{1+x^2}} = (1+x^2)^{-\frac{1}{2}}$ $= 1 + (-\tfrac{1}{2})(x^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(x^2)^2 + \ldots$ $= 1 - \tfrac{1}{2}x^2 + \tfrac{3}{8}x^4 + \ldots$ so $y = (1+x)(1 - \tfrac{1}{2}x^2 + \tfrac{3}{8}x^4 + \ldots)$ $= 1 + x - \tfrac{1}{2}x^2 - \tfrac{1}{2}x^3 + \tfrac{3}{8}x^4 + \tfrac{3}{8}x^5 + \ldots$	M1 A1 M1 A1ft [4] [15]	binomial series with $p = -1/2$ Allow this M1 if $p = ½$. M0 if no working is shown and series is wrong expanding brackets. Ft their previous series

3 (i) $\vec{CA} = -\mathbf{i} - \mathbf{j} - 3\mathbf{k}$, $\vec{CB} = -3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ $\cos\theta = \dfrac{\vec{CA}.\vec{CB}}{\lvert\vec{CA}\rvert.\lvert\vec{CB}\rvert}$	B1	Accept row vectors. Accept the vector equations of CA and CB if they contain the correct vectors **CA** and **CB**
$= \dfrac{(-1).(-3)+(-1).(-1)+(-3).(-2)}{\sqrt{11}.\sqrt{14}}$	M1	Ft their vectors
$= \dfrac{10}{\sqrt{11}.\sqrt{14}} = 0.8058..$	A1	Accept ±0.8058
$\Rightarrow \theta = 36.31°$	A1 [4]	Must be the acute angle. Accept 0.634 radians
(ii) $\mathbf{r.n} = \mathbf{a.n}$ $\Rightarrow x - y + 2z = 2$ when $x = 0, y = 0, z = 1$: $0 - 0 + 2 = 2$ valid	B1 B1 E1 [3]	$x - y + 2z = \ldots$Condone i,j,k. $\ldots = 2$ verifying (0, 0, 1) in plane
(iii) Perpendicular is $\mathbf{r} = 3\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ Meets plane when $(3 + \lambda) - (1 - \lambda) + 6 + 4\lambda = 2$ $\Rightarrow 3 + \lambda - 1 + \lambda + 6 + 4\lambda = 2$ $\Rightarrow 6\lambda = -6$, $\lambda = -1$ Point of intersection is (2, 2, 1) Distance from (3, 1, 3) to (2, 2, 1) is $\sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$	B1 B1 M1 A1 A1ft M1 A1cao [7] [14]	$3\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \ldots$ (Condone omission $\ldots + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ of $\mathbf{r}=$) substituting into equation of plane $\lambda = -1$ (2, 2, 1) distance formula w.w.w. Accept 2.45

4 (i) When $x = 0, y^2 = 1 \Rightarrow y = 1$ or -1 so B is (0, 1) and C is (0, −1) When $y = 0$, $(1 - 2x)^3 = 0$ $\Rightarrow 1 - 2x = 0, \Rightarrow x = ½$ so A is (1/2 , 0)	B1 B1 B1 [3]	$y = 1$ or -1 $x = ½$
(ii) $y^2 = (1 - 2x)^3$ $\Rightarrow 2y\frac{dy}{dx} = 3.(-2)(1-2x)^2$ or $-6+24x-24x^2$ $\Rightarrow \frac{dy}{dx} = -\frac{3(1-2x)^2}{y}$ At P, $x = 0, y = 1, \Rightarrow \frac{dy}{dx} = -\frac{3.1^2}{1} = -3$ *	M1 M1 A1 M1 E1	$2y\frac{dy}{dx} =$ Chain rule $3.(-2)(1-2x)^2$ or expansion and differentiation Correct result. Unsimplified will do. Substituting $x = 0$, $y=1$ www.
or $y = (1 - 2x)^{3/2}$ $\Rightarrow \frac{dy}{dx} = (3/2)(1-2x)^{1/2}(-2)$ $= -3(1-2x)^{1/2}$ When $x = 0$, $\frac{dy}{dx} = -3$ *	M1 M1 A1 M1 E1 [5]	Chain rule Correct result, unsimplified will do Substituting $x = 0$ into their dy/dx www
(iii) $y = t^3 \Rightarrow t = y^{1/3}$ $\Rightarrow x = \frac{1}{2}(1-t^2) = \frac{1}{2}(1-y^{2/3})$ $\Rightarrow 2x = 1 - y^{2/3}$ $\Rightarrow y^{2/3} = 1 - 2x$ $\Rightarrow y^2 = (1 - 2x)^3$ * <u>Or</u> $y^2= (t^3)^3$, $(1-2x)^3=(1-2(1/2(1-t^2)))^3$ $= t^6$ $= t^6$ $\Rightarrow$ $y^2=(1-2x)^3$	M1 A1 E1 M1, A1 E1www	Attempt to eliminate t any valid relationship between x and y www Substituting parametric coordinates into the LHS and RHS of the equation, correct expressions unsimplified
$\frac{dy}{dx} = \frac{3t^2}{-\frac{1}{2}.2t} = -3t$ Equation of tangent: $y - y_1 = m(x - x_1)$ $\Rightarrow y - t^3 = -3t\,[x - \frac{1}{2}(1-t^2)\,]$ $= -3tx + \frac{3}{2}t - \frac{3}{2}t^3$ $\Rightarrow 2y - 2t^3 = -6tx + 3t - 3t^3$ $\Rightarrow 6tx + 2y + t^3 - 3t = 0$ *	M1 A1cao M1 M1 E1 (5)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $-3t$ using $y - y_1 = m(x - x_1)$, or finding c in $y=mx+c$ after substituting $x_1 = \frac{1}{2}(1-t^2)$, $y_1 = t^3$ www

	[8] [16]	

Examiner’s Report

Pure Mathematics 3 (2603)

General Comments

Candidates performed remarkably well on this paper with a very high proportion scoring 60 marks or mor out of the 75 marks available. Presentation was generally good, candidates appeared to be confident applying the techniques required and often obtained the solutions to the questions with a minimum working. There was little evidence of candidates being short of time; most completed the four questions Section A, and work crossed out was usually replaced by a correct version. It was, perhaps, significant th there was much less evidence than in recent papers of poor algebra.

Comments on Individual Questions

Question 1 (Various)

(a) Most candidates were familiar with integration by parts and chose the correct order of terms; the on error that was at all common was an error in the sign of $\int \cos x \, dx$.

(b) This was one question which was not well done except by the stronger candidates. Others were unable to apply the chain rule with $u = \sin 2x$ and $y = u^3$, sometimes choosing, instead, to put $u = 2x$. A number of candidates resorted to the product rule with $u = \sin 2x$ and $v = \sin^2 2x$, which, of course, still required the chain rule unless the double angle formula was used. Unfortunately for those attempting this approach, $\sin^2 2x$ was often written as $\frac{1}{2}(1-\cos 2x)$ instead of $\frac{1}{2}(1-\cos 4x)$.

(c) This question was most often done correctly, but some candidates, who did not write down the identity, got $\sin\alpha$ and $\cos\alpha$ the wrong way round. A few candidates wrote $\sin\alpha = -3$.

(d) Apart from some cases of rather doubtful cancelling, this question was well done. Most candidates obtained the correct limit $\cos x$, but very few indeed gave this as the derivative of $\sin x$.

[(a) 1; (b) $6\sin^2 2x\cos 2x$; (c) 3.6, 56.3°; (d) $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$]

Question 2 (Partial fractions, differential equations and the binomial theorem)

(i) There was a very good start to this question with most candidates obtaining the correct partial fractions. A rare careless error was $2 = 2A \Rightarrow A = 2$, and sometimes candidates who obtained A, B and C correctly wrote down the partial fractions as $\frac{1}{1+x} - \frac{1}{1+x^2}$.

(ii) Many candidates were able to separate the variables of the differential equation and perform the correct integrations. Occasionally $\int \frac{x}{1+x^2}dx$ was given as $2\ln(1+x^2)$, and, for those who made the error in (i) above, $\int \frac{1}{1+x^2}dx$ was almost always written as $\frac{1}{2}\ln(1+x^2)$.
The final stages of this question were not generally well done. Candidates who found the constant of integration before any attempt to simplify the logarithms usually did so correctly, but those who chose to simplify their expression, with the constant still present, most often made errors in doing so. Most common was $\ln y = \ln(1+x) - \frac{1}{2}\ln(1+x^2) + c \Rightarrow y = \frac{1+x}{\sqrt{1+x^2}} + c$, or, perhaps, the above line but with e^c or A instead of the final c.

(iii) Apart from a few candidates who expanded $(1+x^2)^{1/2}$ or $(1+x^2)^{-1}$, this part was nearly always answered correctly.

[(i) $A = 1$, $B = -1$, $C = 0$; (iii) $1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 + \ldots$, $1 + x - \frac{1}{2}x^2 - \frac{1}{2}x^3 + \frac{3}{8}x^4 + \frac{3}{8}x^5 + \ldots$]

Question 3 (Vectors)

Most candidates knew how to find the angle between two vectors and full marks were common. Infrequent errors included finding **AC** and **BC** instead of **CA** and **CB**, and using $\mathbf{a}\,.\,\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta$.

(ii) Again, there was a very pleasing number of completely correct solutions but a small number of candidates, who chose to start with $ax + by + cz + d = 0$ or $ax + by + cz = d$ and then to find d using the scalar product **a.n**, were often confused about the sign of d and clearly had to make changes in order to show that the point B lay in the plane. A significant minority of candidates confused the equations of a line and a plane.

(iii) This question was also well answered and there were very few careless errors. Some candidates obtained the final result by using the formula for the distance of a point from a plane, rather than the method suggested in the question.

[(i) $-\mathbf{i}-\mathbf{j}-3\mathbf{k}$, $-3\mathbf{i}-\mathbf{j}-2\mathbf{k}$, 36.3°; (ii) $x-y+2z=2$;
(iii) $\boldsymbol{r}=3\mathbf{i}+\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+2\mathbf{k})$, (2, 2, 1), $\sqrt{6}$]

Question 4 (Coordinate geometry, parametric equations)

Full marks for this question were common reflecting a pleasing strength in handling the algebra.

(i) This was usually answered correctly although sometimes with forms such as $B = 1$, or $\sqrt{1}$ in the answer.

(ii) Both implicit and explicit differentiation were used correctly with very few errors.

(iii) Generally well answered although in some cases methods were somewhat confused. However the essential steps were usually present. There were occasional sign errors in expanding $(1-(1-t^2))^3$. Also, very occasionally, candidates verified the results for a particular point only, instead of generally.

Candidates usually had no difficulty in finding the value of $\frac{dy}{dx}$ in terms of t, but a few left the result as $\frac{3t^2}{-t}$.

Those candidates who used the equation $y - y' = m(x - x')$ most often obtained the equation of the tangent correctly but those who used the form $y = mx + c$ sometimes failed to find the value of c after substituting the coordinates $(\frac{1}{2}(1-t^2), t^3)$.

[(i) $(\frac{1}{2}, 0)$, $(0,1)$, $(0,-1)$; (iii) $\frac{dy}{dx} = -3t$]

Section B (Comprehension)

Most candidates scored ten marks or more on this section and there was a good spread of marks between 10 and 15.

Question 1 Answered by all candidates but quite a large number gave only a verbal argument comparing a circle to a circle.

Question 2 A small number of candidates placed Neptune in the gap in Table 2 giving it aBode's number of 42. Otherwise candidates usually did this question correctly.

Question 3 A few candidates misunderstood the question and gave the first term of the G.P. as 1, but almost all candidates used their first term correctly, making the correct deduction where possible.

Question 4 Two errors were made in this question. Fairly common was the use of $a = 7.4$ or, less common, the failure to take the square root to find the value of e.

Question 5 This question was the least well done of the five questions, many candidates being rather confused by it. Common errors were :-

$$R_1 = \frac{2}{3} \times 5.8, \text{ or } \frac{3}{2} \times 6,$$

$$R_n = \frac{3}{2} \times (6+4.5\times2^{n-2}), \text{ or } 4+3\times\left(\frac{4}{3}\right)^{n-2}.$$

Some candidates left their expressions unsimplified, e.g. $R_1 = \frac{6}{1.5}$.

Question 6 Most candidates were able to make the correct choice of moons, but quite a large number failed to show sufficient method to achieve both marks available.

Question 7 There was a wide variety of answers to this question and, although the majority of candidates included the appropriate facts and achieved the mark, many cast some doubt on their understanding by including irrelevant arguments about the underlying physics or referring to newly formed planets. A few candidates missed the point entirely and referred only to our own solar system.

[**3.** ½, 8.25×10^7; **4.** 0.258; **5.** 4, $4 + 3 \times 2^{n-2}$; **6.** Miranda, Umbriel and Oberon]